

Random number generation with cosmic photons

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Abstract

Random numbers are used in a broad spectrum of scientific researches and industrial applications ranging from physics foundation tests to information encryption. In particular, nonlocality tests provide a strong evidence to our current understanding of the nature — quantum mechanics. All the random number generators used for the existing tests are constructed locally, which are essentially based on quantum mechanics principles. Using quantum random numbers to test quantum mechanics may introduce a loophole to these foundation tests, which is called randomness loophole or freedom-of-choice loophole. Here, we report an experimental realization of random number generation based on the arrival time of photons from a number of cosmic sources. The measurement outcomes (raw data) pass the standard NIST statistical test suite. Furthermore, we design a scheme to use these RNGs in Bell's inequality tests, which shall address the randomness loophole.

Introduction.— Randomness is one of the most fundamental features of nature. The best example may be the biological diversity[1]. Another example is the Brownian motion[2, 3] which has been familiar to us for nearly two centuries. Random number generators (RNG) based on either a classical mechanism or a quantum process are indispensable in our society for a variety of applications. Quantum random number generators (QRNGs) rely on breaking quantum superpositions results in unpredictable measurement outcome and are therefore deemed to possess the true randomness. A number of quantum processes are utilized to make QRNGs (for reviews see Ref.[4, 5] and references therein). Laser-based practical QRNGs have advantages of operational simplicity and high bit rates, which are desirable properties in realistic applications.

Bell tests, or experimental violation of Bell’s inequality, provide a strong support to the theory of quantum mechanics, especially to rule out the local hidden variable models. Recently, both locality and efficiency loopholes are closed in Bell test experiments[6–8]. In these tests, QRNGs are employed in two remote test sites. However, the test results are not reliable if the two random number generators are somehow correlated. This is called randomness loophole (also known as freedom-of-choice loophole). The randomness loophole may be asymptotically closed by adopting the RNG scheme based on cosmic photon measurements, which attempts to take the advantage of the randomness born at the remote celestial object[9]. The underlying assumption is that the photons from two distant cosmic radiation sources are not correlated and contain certain randomness. Such assumption is reasonable for the ultimate test of quantum mechanics.

In this paper, we present the experimental realization of RNGs based on the arrival time of photons from a number of cosmic sources, with magnitude (a measure of the brightness of a celestial object seen on Earth) between 4.85 and 12.45 and distance (from Earth) between 3706 and 7.49×10^8 light years (ly). The RNGs can deliver raw random data at a rate exceeding $10^6 s^{-1}$. Without hashing, the streams of random bits have close to ideal min-entropy and pass NIST statistical test suite[10], thereby certifying the quality of these cosmic photon-based RNGs. These RNGs possess a unique advantage that the originality of the randomness is space-likely separated from human activities on Earth, making them particularly useful for causality-strict applications. As an application example, we present a realistic analysis to show that one can eliminate the concern of local correlations occurring in the past sixteen thousand years in an event-ready test of Bell’s inequality by using RNGs

with photons from cosmic radiation sources with magnitude 9. Furthermore, we show that it is experimentally feasible to conduct the test of Bell's inequality with RNGs made with photons from quasars of high redshifts, which may be the ultimate one for this type of test to reject any local hidden variable mechanism in the frame of relativity.

Experiment on random number generation with cosmic photons.— We conduct an experimental study of generating random numbers with cosmic photons in the Astronomy Observatory at Xinglong, China (N 40°23.75', E 117°34.5'). We use a Ritchey-Chretien (RC) optical telescope with a diameter of 1 meter and a focal length of $f = 5$ meter to collect the light from the cosmic radiation source under study (CRSS) and use prisms to direct lights of different spectral bands to different applications. The light that is incident onto this RC telescope from a typical cosmic radiation source with an angular spread of $\phi = 3''$ has an 1/e-diameter of $73 \mu m$ and a numerical aperture (NA) of 0.10 at the focal plane. A multimode optical fiber with NA = 0.22 and a core diameter of $105 \mu m$ is placed at the focal plane to collect the light with wavelength in the range, [680 nm, 830 nm], and direct the light to a single photon avalanche diode (SPAD, model: EXCELITAS, active area: $170 \mu m$, single photon detection efficiency: $\sim 55\%$ at 780 nm). A CCD camera is also placed at the focal plane to detect the light with wavelength in the range, [530 nm, 680 nm]. The formed focal image of the CRSS is used in a tracking-locking mechanism to stabilize the coupling of cosmic photons from the CRSS into the multimode fiber. The rest part of the cosmic light is used for astronomy applications. We estimate the transmittance of the entire optical system to be about $20.7\% = 53\%$ (transmittance of the RC telescope) $\times 75\%$ (transmittance of the prisms) $\times 52\%$ (efficiency to coupling light into the multimode fiber). We estimate the transmittance of light through the atmosphere to be about 20%. As a result, we estimate the total efficiency to be about 2% to detect a cosmic photon from the CRSS. (It should be noted that this efficiency is estimated with static condition and may vary, e.g., the atmospheric disturbance may deteriorate the coupling efficiency by displacing the focal image of the CRSS off the multimode fiber.)

The state of the single photons emitted by the CRSS may be described as an incoherent mixture of pure states, and each pure state is a coherent superposition of orthonormal states in the time domain, the use of the latter property leads to the creation of laser-based QRNGs[11–18]. The incoherent mixture results in Poissonian photon statistics with a feature that the mean photon number is a constant for equal time slots and the time interval

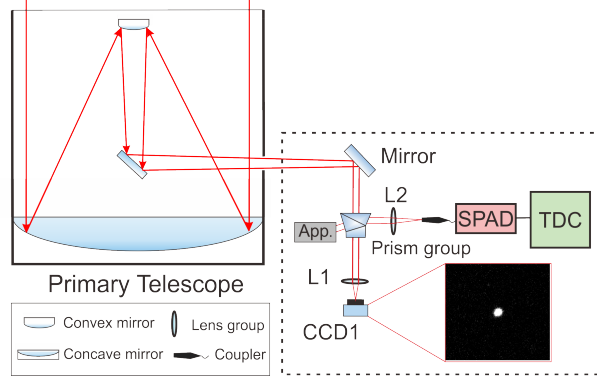


FIG. 1: Random number generation with cosmic photons. The photons from a cosmic radiation source under study (CRSS) incident onto the telescope are collected by a set of reflective mirrors. The photons with wavelength in the range, [680 nm, 830 nm], are reflected by a prism group, coupled into a multimode optical fiber, and detected by a single photon avalanche diode (SPAD). The SPAD output is sent to a time-to-digital converter (TDC). Inset: The photons with wavelength in the range, [530 nm, 680 nm], form an image of the CRSS, here, TYC 2354-243-1 on the camera, whose profile is used to track the CRSS and to lock the coupling of photons into the multimode fiber. The photons with other wavelength are used for astronomy applications (APP.).

between emitted single photons is random. If dividing the period T_W of a reference clock equally into N time bins, the probability for a cosmic photon to arrive at an arbitrary time bin t_i ($i = 1, 2, \dots, N$) is a constant, $P_i = 1/N$.

The photon-detection signal from the SPAD is recorded using a home-made time-to-digital converter (TDC) with a time resolution of 25 ps. We set $T_W = 40.96$ ns which is smaller than the recovery time (45 ns) of the SPAD such that there is at most one detection event per clock cycle. By dividing each clock cycle T_W into $N = 256$ ($\times 160$ ps) time bins and labelling each time bin with an 8-bit binary code accordingly, the 8-bit binary code is recorded for the time bin in which a detection event occurs.

We examine the statistics of random numbers based on photon arrival time for a number of stars and quasars. First, we notice that the photon counting signal rates exceed $10^6 s^{-1}$ (which is within the linear operation mode of the SPAD in use) for CRSS with smaller magnitude as shown in Table I, proving that this method is as efficient as the laser-based RNGs in generating random numbers. Second, despite the dramatic fluctuation of signal rate (shown by the signal ranges in Table I) for a CRSS and between CRSS with similar

TABLE I: Photon counting data for cosmic radiation sources under study (CRSS)[19–21]

Name	Magnitude	Distance	Signal rate	total Data	background	r	min-entropy
		(ly)	($\times 10^6 s^{-1}$)	(Gb)	(s^{-1})		H
HIP15416	4.85	1177.46	2.20-2.28	1	914	2450	0.9969
HIP117447	5.43	16300	0.86-1.2	1	512	2012	0.9978
HIP2876	5.75	3623.96	0.51-0.53	1	464	1130	0.9981
HIP6522	6.07	32600	0.48-0.68	1	578	1010	0.9983
HIP3030	6.75	5346.83	0.64-0.65	1	518	1260	0.9976
HIP100548	7.03	40750	0.33-0.52	1	615	680	0.9973
HD33339	7.99		0.23-0.24	1	662	350	0.9980
HIP20276	8.24	65200	0.18-0.26	1	486	400	0.9980
HIP3752	9.02	4076.95	0.12-0.13	1	532	235	0.9973
HIP114579	9.27	163000	0.05-0.10	1	442	170	0.9974
HIP117690	9.9	163000	0.33-0.43	1	674	57	0.9938
HIP23114	10.6	163000	0.009-0.013	0.1	417	28	0.9909
TYC 2354-243-1	12.45	7.49×10^8	0.0011-0.0031	0.1	359	8	0.9897
2MASX							
J00455809+3933494	15	7.76×10^8	0.00067-0.00077	0.02	617	1.2	0.9822

magnitudes, the true signal rate (with background subtracted) generally scales with the magnitude as expected, as indicated by the linear trend line in the logarithm plot of Fig. 2. We attribute the fluctuation in the rate partly to the atmospheric disturbance. Besides attenuating the cosmic light, the atmospheric disturbance may seriously deteriorate the coupling of light from the CRSS into the multimode fiber by either displacing the focal position or modifying the beam profile, which cannot be compensated by the locking mechanism. The limited locking bandwidth also decreases the efficiency to couple cosmic photons into the fibre. We use the maximum rate for each CRSS to minimize the impact due to disturbance and do not use the data of quasars because of their small statistics in generating the trend line. We notice that the data for quasar:TYC 2354-243-1 (red filled dots) and quasar: 2MASX J00455809+3933494 (blue open dots) are much below the trend line, which may be partly due to the disturbance. Another reason may be that the radiation intensity of the

two quasars have decreased significantly.

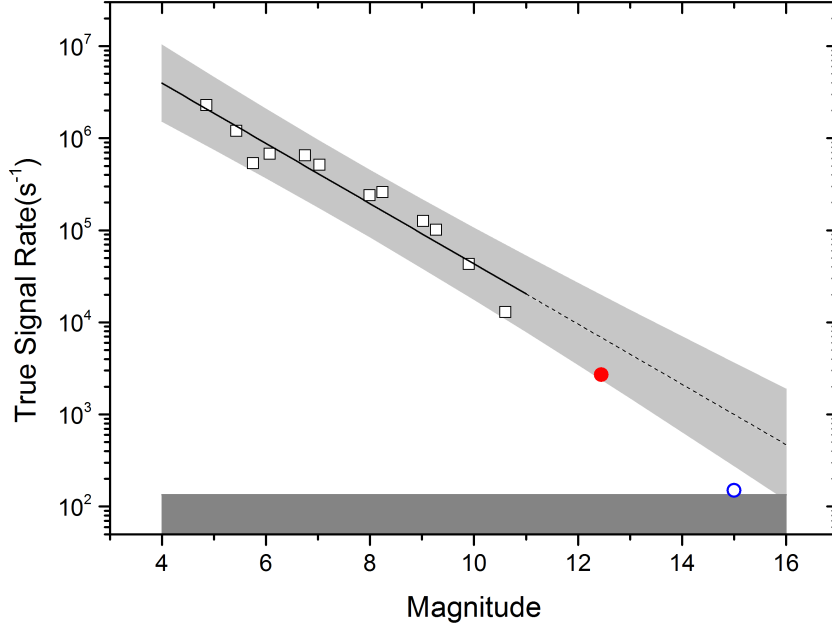


FIG. 2: Experimental true signal rate (with background subtracted) versus magnitude. The trend line (solid) is fitted with data (open square) for CRSS with magnitude < 11 . The trend line is extrapolated to magnitude 16 (dashed line). The horizontal line is the averaged background. The shades stand for 2-standard deviation. Red filled dot: data for quasar TYC 2354-243-1; blue open dot: data for quasar 2MASX J00455809+3933494.

We observe that for each CRSS, the probability (P_i) of photon arrival time uniformly distributes around the ideal value of $1/256$, as shown in Fig. 3, indicating a good level of randomness. We then apply two standard methods to evaluate the performance of cosmic photon-based RNGs. First, the min-entropy, $H_\infty = -\log(\max P_i)$, of the raw data with the signal-to-noise ratio of > 100 is consistent with the ideal value of one within 0.3%, and is still within 1% even when the signal-to-noise ratio drops to 8 for the raw data obtained with quasar: TYC 2354-243-1 (with magnitude 12.45). Second, the raw data obtained with stars pass the NIST statistical test suite[10] (see appendices). (The data for quasars fail the test, partly because of small statistics and large background contribution. Fig. 2 shows that it is hard to distinguish signals (blue open dots) from noise for quasar 2MASX J00455809+3933494.) These two results certify the quality of randomness of the data stream

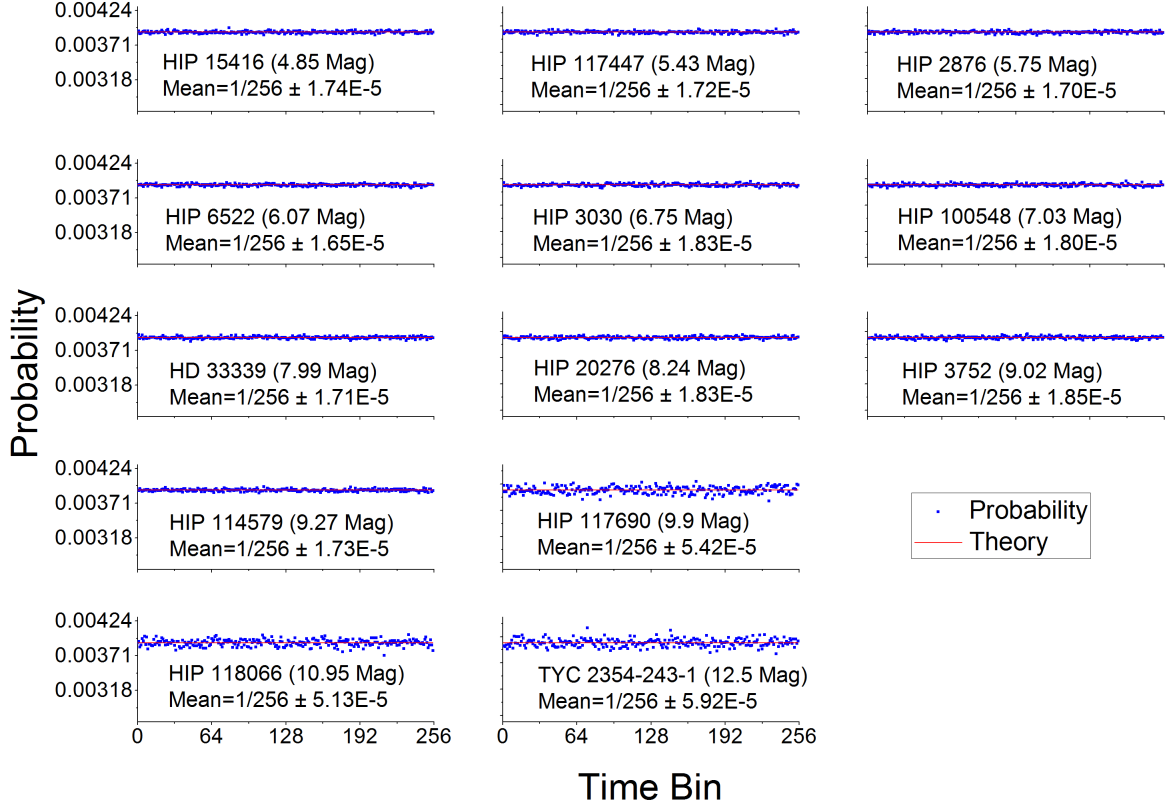


FIG. 3: Experimental probability distribution (scatter dots) of photo-detection events versus time bin. Smooth line (red) is the theoretical value $= 1/256$.

from these RNGs. Because the radiation intensity of a cosmic source is inversely proportional to the magnitude, we would expect to generate random bits at a rate exceeding $10^7 s^{-1}$ and with much high signal-to-noise ratio by preparing RNGs with CRSS with smaller magnitudes. Such cosmic-photon based RNGs deliver random bits at high rates without the need of hashing and with the distinct advantage that the originality of the randomness is space-likely separated from any events occurring on Earth are particularly useful in causality-strict applications.

To estimate the background contribution, which includes both stray light-induced photon-detection events and detector dark counts that are indistinguishable with the events induced by the photons from the CRSS, we orient the RC telescope slightly from its aligned angle (for best collection of cosmic photons) till the photon counting rate drops to a stable level,

we then suppress the detection events induced by photons from the CRSS while keeping the background unchanged. We notice that the background-induced signal rates are at the level of 500 s^{-1} , which may be contributed by local sources, e.g., the detector dark counts and scattered light from ground-based sources. The locally generated random numbers contrast our proposal of cosmic photon-based RNGs to utilize the randomness born at the remote cosmic radiation source. These locally generated bits may be used by local hidden variable theories and therefore must be made insignificant as indicated by big signal-to-noise ratios. Below we present a realistic analysis on the test of Bell's inequality with cosmic photon-based RNGs which have large signal-to-noise ratio.

Test of Bell's inequality with cosmic-photon based RNGs— The celebrated Bell's inequality[22, 23] is based on the assumption of locality, realism and freedom-of-choice. In previous Bell experiments[6–8] with a pair of entangled particles A and B, the events for preparing a pair of entangled particles A and B, setting the measurement base for A (B), and measuring the state of A (B) are kept space-likely apart in the future light cones. However, the light cones of these events cross each other in less than 1 ms in the past direction, with the experimental setup including the RNGs to set the base choice distributed within a scope of 1 km. It is foreseeable that a local correlation event occurred in the overlapped region can deterministically control the measurement results. Furthermore, it was shown that a Bell experiment is vulnerable to the local hidden variable theory even with a conspiracy of as little as 1/22 bit of mutual information between the RNGs and the source of entangled particles[24]. Here we consider three possible scenarios that local correlation events may impact the measurement results in a Bell experiment, which are shown in Fig. 4. In the first case, a local correlation event Y1 may inform the source before state preparation about the photon emission event S1 for random bit generation, (denoted by the local hidden variable λ_1), so does a similar event Y2 (denoted by the local hidden variable λ_2). In this case, the source has full information about the base settings denoted by local hidden variables (λ_1, λ_2) prior to state preparation, provided that the local correlations (λ_1, λ_2) take place by an amount of time, $\tau \geq \max(L_1/c, L_2/c)$ back into the history, where L_1, L_2 are the distances of the two cosmic radiation sources from Earth. For example, if we choose HIP 114579 (magnitude: 9.27, distance: 163000 lys, N $28^\circ 34' 37.5''$ E $23h12m41.10s$) and HIP 38520 (magnitude: 8.64, distance: 19000 lys, N $-15^\circ 51' 45.5''$ E $7h53m19.09s$) as the cosmic photon sources for RNGs (which has a signal-to-noise ratio of $\sim 170 \gg 22$), the Bell

experiment is immune to local correlation events occurring as early as 163000 years ago, which is a great advancement, comparing to less than 1 ms in previous Bell experiments. In the second case, if we replace one cosmic photon-based RNG with a locally prepared RNG in the Bell experiment, we have a similar result that the local correlation event must occur before state preparation by $\tau \geq L_{1(2)}/c$ to impact the measurement results. In the third case, the local correlation event (denoted by a hidden variable λ_3) may occur in the overlapped region formed by the past light cones of two cosmic photon radiation events, S1 and S2 with a time advancement of $\tau \geq (L_1 + L_2 + L_{12})/2c$, which is about 181000 years in this example, where $L_{12} \sim 180000$ lys is the separation between HIP 114579 and HIP 38520.

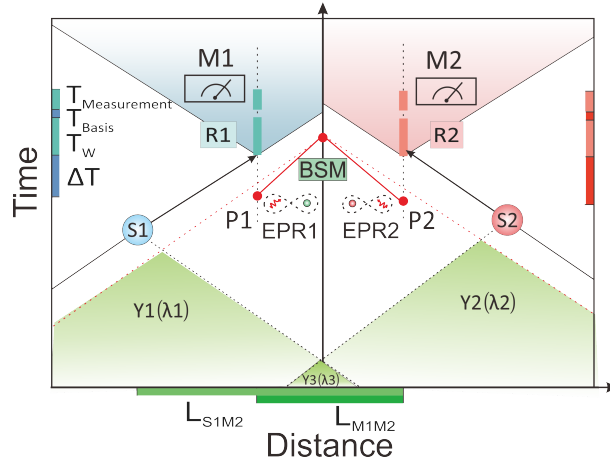


FIG. 4: Space-time diagram of an event-ready Bell experiment with NV-centers [6, 9]. S1 and S2 are cosmic photon emission events, followed by events R1 and R2 to output random bits for base setting. P1 and P2 are events to send photons for Bell state measurement (BSM) after the creation of entangled photon-electron pairs at NV centers. A destructive BSM with photons from the two entangled electron-photon pairs prepares the two electrons in a Bell state, which are ready for state measurements (M1 and M2) after the base setting. Y1, Y2 and Y3 are local correlation events (denoted by local hidden variables, λ_1 , λ_2 , λ_3 , respectively,) that may occur in the overlapped region formed by the past light cones. ΔT is the time elapsed between sending the photon (entangled with the electron) for BSM and that the reference clock starts a new cycle. It is set to keep the events of cosmic photon emission and BSM space-likely separated. T_{Basis} is the time elapsed for the event completing the base setting upon receiving a random bit, and $T_{Measurement}$ is the time elapsed for the event completing the state measurement after the base setting.

The two RNGs discussed in the above output random bits to set bases at an average

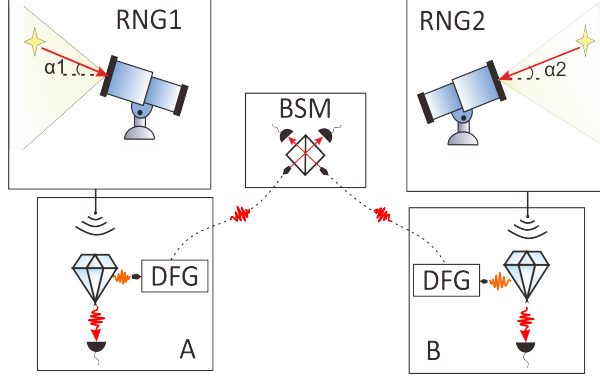


FIG. 5: Schematic of an event-ready Bell experiment. Each measurement station (A or B) prepares an entangled electron-photon pair with a NV center. A and B downconvert the single photons at the visible to photons at the wavelength of 1550 nm via difference frequency generation (DFG). A successful BSM with a single photon from A and a single photon from B projects the corresponding two electrons into a Bell state. RNG1 and RNG2 provide random bits per time window T_W to set the base in measuring the quantum state of the electron spin. α_1 and α_2 are angles of the optical axes of telescopes with respect to the horizon.

rate of $7.5 \times 10^4 (s^{-1})$. We now discuss using them in an event-ready Bell experiment with NV centers[6], as shown in Fig. 5. We conveniently choose the time window (period of the external clock) to be $T_W \sim 14 \mu s$. The probability for the two RNGs to output a pair of random bits simultaneously is $P_{RNG} = (1 - e^{-1})^2 \sim 0.4$.

The distance L_{M1M2} between the two measurement stations A and B is determined according to the requirement of space-like separation between the events of measurement outcome and base setting, $T_W + T_{Basis} + T_{Measurement} + L_{S1M1}/c < L_{S1M2}/c$, where T_{Basis} is the time elapsed for events completing the base setting upon receiving a random bit, $T_{Measurement}$ is the time elapsed for events completing the state measurement after the base setting (see Fig. 4), and $L_{S1M1}(L_{S1M2})$ is the distance between the cosmic source and the first (second) measurement station. With a little math, we have $L_{M1M2} > ((T_W + T_{Basis} + T_{Measurement}) \cdot c) / \cos \alpha$. Taking $T_{Measurement} \sim 4 \mu s$, $T_{Basis} < 1 \mu s$ and the elevation angle $\alpha \leq 30^\circ$ for each telescope, we have, $L_{M1M2} > 6.6$ km. Considering the preparation of event-ready entanglement with two NV centers, according to the requirement of space-like separation between the events of cosmic photon emission and Bell state measurement, the photon-electron entanglement is created before the reference clock

starting a new cycle by $\Delta T > (3/2 - \cos \alpha) \cdot L_{M1M2}/c \sim 14 \mu s$. So the photon-electron entanglement needs to be prepared $\Delta T + T_W + T_{Basis} \sim 29 \mu s$ before the state measurement, which is much smaller than the coherence time ($T_2 \sim 0.6s$ at 77 K[25]) of the electron spin of NV center.

The photons emitted by NV centers at the visible wavelength (~ 640 nm) are subject to high propagating loss in the optical fiber and are downconverted to photons at the wavelength of ~ 1550 nm via difference frequency generation (DFG). The estimated overall photon loss includes about 1.5 dB for downconversion operation[26], 1.4 dB for photon propagation over the 6.6 km optical fiber, 3 dB in the two-photon Bell state measurement, and 2 dB in photon detection. Together with the efficiency in the creation of photon-electron entanglement, the total success probability per entanglement generation attempt is estimated to be $P_{total} = 1.34 \times 10^{-8}$. This will result in one event-ready entanglement per 0.78 hours per $38 \mu s$ measurement period. Therefore, it will take about 193 hours to accumulate experimental data to violate the Bell's inequality with statistical confidence similar to what was achieved by Hensen et al.[6], but with an improvement that the experiment is not affected by local correlation events occurring as early as ~ 180000 years ago.

Discussions.— Following the above discussion, an arising question is whether one can asymptotically close the randomness loophole in a Bell test. Below we examine the recent proposal of testing Bell's inequality rejecting local hidden variable mechanisms from the time of Big Bang with RNGs made with photons from quasars of high redshift, such as APM 08279+5255 with magnitude 15.3 and redshift $z = 3.91$ [27], based on our realistic experimental results.

We notice that the signal rates and signal-to-noise ratios are low for CRSS with high magnitude. The low signal-to-noise ratio is favored by the local hidden variable theorists. We perform an analysis on the root cause of the background signals. We find that the detector dark counts are not more than $1 s^{-1}$ [28] and the laboratory background contributes about $10 s^{-1}$ on average. So the primary noise is due to the sky brightness, which is in the range of $[16, 22]$ per $arcsec^2$ (measured by solid angle) at Xinglong[29], corresponding to radiations with magnitude in the range of $[13, 19]$ for our current measurement system. In our experiment, the average detected background contribution is $560 s^{-1}$ according to the data in Table I, which corresponds to the radiation with magnitude 15.8. We attempt the measurement with quasar TYC 2354-243-1 (with magnitude 12.45) and quasar 2MASX

J00455809+3933494 (with magnitude 15). The data for the former one is below the trend line by about two standard deviation, which could be a result of disturbance. The data for the latter one is much below the trend line, which may be due to a significant change in the radiation intensity. (It should be noted that the image of a CRSS can be obtained by performing a long time integration to defeat the noise while signal is recorded instantly.) Based on the trend line, we project the signal rate to be 850 s^{-1} for APM 08279+5255 with magnitude 15.3, with the background contribution at the rate of 50 s^{-1} when Xinglong has the best background (for magnitude 19). The signal-to-noise ratio of 17 is much below the threshold of 22, which is insufficient to conduct the proposed experiment.

A review of the system shows that the telescope system with 1 meter aperture and $f = 5$ meter was designed to have a beam waist of $9.2 \text{ } \mu\text{m}$ and a field-of-view of $0.4''$ in the diffraction limit at the central wavelength of 755 nm . The parameters would become $44 \text{ } \mu\text{m}$ and $1.8''$ due to the local atmospheric lens effect at Xinglong[29]. One can use the optical fiber with core diameter of $50 \text{ } \mu\text{m}$ to efficiently collect photons from the CRSS. The background due to the sky brightness will be equivalent to the radiation with magnitude 20.7. This will result in a background signal rate of 14 s^{-1} and a signal-to-noise ratio of 58, which is sufficient for the proposed experiment. However, we only have a beam waist of $70 \text{ } \mu\text{m}$ and a field-of-view of 14.7 arcsec^2 for the current system in reality and therefore have a poor signal-to-noise ratio due to collecting background in a much larger solid angle. This can be fixed by replacing the prisms with bandpass filters, because prisms may distort the beam profile seriously. In addition, if we use adaptive mirrors to replace current mirrors in the telescope, the final beam waist and the field-of-view can be made close to diffraction free, which will improve the signal-to-noise ratio to be > 200 [30]. Furthermore, the signal-to-noise ratio can be further improved by designing a telescope system with a smaller field-of-view. As such, based on our experimental measurements and the systematic analysis, we conclude that the proposed Bell test with quasars of high redshifts to reject local hidden variable models from the time of Big Bang is within the reach.

Conclusion. — In conclusion, we demonstrate the generation of RNGs based on the arrival time of photons from a number of cosmic radiation sources, which produce raw random bits at high rates, having close to ideal min-entropy and passing the NIST statistical test suite. In particular, these RNGs possess a distinct feature that the randomness is born at the remote celestial object, making them particularly useful in causality-strict applications. We

present a design to apply such RNGs in an event ready Bell test. Our analysis shows that it is feasible to conduct the test of Bell's inequality with RNGs made with photons from quasars of high red shift, which may be the ultimate one for this type of test to reject any local hidden variable mechanism in the frame of relativity. Meanwhile, our single-photon detection based measurement setting also provide a powerful tool for cosmology observation.

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